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CALCULATION OF AN EJECTION EXPLOSION IN A RADIAL APPROXIMATION

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UDC 539.3

Equations are derived describing the motion of a medium with an ejection explosion, under the assumption that the medium is incompressible and moves in a radial direction away from the center of the explosion. Here account is taken of the tangential stresses between the moving layers of the medium. A comparison of the calculations of the velocities of the motion of the dome and the dimensions of the craters formed showed good agreement, both with model experiments on the ejection of sand, and with large-scale ejection explosions.

1. Introduction

The development of an ejection explosion in soil or rock with time can be represented in the form of three basic stages [1, 2]. The underground-explosion stage lasts from the moments of the detonation of the charge up to the arrival of the wave at the surface. Here the motion of the medium is close to spherical symmetry. In the second stage, starting after reflection of the wave from the free surface, a dome develops. This stage continues up to the moment of the breakthrough of the gases from the cavity to the atmosphere. After this, the dome breaks down rapidly and, during the succeeding moments of time, in the third stage, there is a ballistic dispersion of particles between which there is very little connection.

The two-dimensional, not fully established motion of the medium in the second stage determines to a considerable degree the dimensions of the future crater. A complete investigation of this motion is complicated and is possible only using high-speed computers. In [3-6], methods are proposed for calculating the equations of an elastoplastic medium with two spatial variables. Such calculations require a large amount of machine time; therefore, they are not very suitable in cases where a large number of variants is needed for the analysis.

To make preliminary calculations aimed at clarifying the effect of the parameters characterizing the properties of the medium and the conditions of the conduct of the explosion on the ejection crater, it is expedient to use less complicated methods, requiring a small amount of machine time for each variant. In the construction of a simple ejection model, in the present work the following assumptions are used: 1) the motion of the medium in the second stage takes place only in a radial direction; 2) the medium is incompressible.

The first postulation is based on the fact that the development of the dome starts after the end of the spherically symmetrical underground-explosion stage, in which the velocity has a radial direction.

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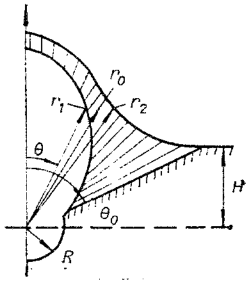

Fig. 1

In spite of the fact that, after reflection of the wave from the free surface, the velocity changes direction somewhat, it subsequently again becomes close to radial, since the change in the stresses is mainly determined by the pressure drop between the cavity and the free surface and takes place (approximately) in a radial direction.

On the photographs given in [7] it can be seen that, in the stage of the formation of the dome, the motion of a weakly connected soil is close to radial. In addition to this, experiments with the scattering of radioactive pickups [8] show that particles of the soil which before the explosion lay on exactly the same radius, drawn from the center of the explosive to the surface, fall at exactly the same place. This important special characteristic of the motion of a medium with ejection shows that, with calculations in a radial direction, it is sufficient to use only one calculating cell.

The assumption of incompressibility is based on the fact that, with optimal depths of the charges, the stresses in the soil at the moment of the arrival of the wave at the surface of the ground are small and do not bring about any significant compression.

The velocity assumed by the medium under the action of these stresses is taken into consideration in the present work by the fact that, in calculations of the ejection in the second stage, the medium has an initial velocity in the radial direction.

A model of an incompressible elastoplastic medium was used in [9] with consideration of the problem of the spherically symmetrical explosion in the ground.

We take note of [10], which discussed a simple model of an ejection explosion. However, in the model of the medium considered here there was no friction between adjacent elements, and, in the equation of motion of the incompressible medium, no account was taken of the force of lateral thrust. Agreement with the ejection experiment was achieved by selection of the coefficient of the resistance force, introduced in [10], as proportional to the velocity of an element of the medium.

2. The Equation of Motion

In the spherical system of coordinates r, θ, φ (correspondingly, the radius, the polar angle, the longitude) the equation of motion of a continuous medium for the radial direction has the form

$$\rho \frac{\partial v_r}{\partial t} = \frac{1}{r^2}\frac{\partial (r^2\sigma_r)}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial (\tau_{r\theta}\sin\theta)}{\partial \theta} - \frac{\sigma_\theta + \sigma_\varphi}{r} - \rho g \cos\theta, \tag{2.1}$$

where ρ is the density of the medium; t is the time; v_r is the velocity in the radial direction; σ_r, σ_θ, σ_φ are the normal stresses; $\tau_{r\theta}$ is the tangential stress; g is the acceleration due to gravity. In the equation it is taken into consideration that the motion of the medium has symmetry around the axis $\theta=0$.

We assume that the motion takes place only in the radial direction, and that the medium is incompressible,

$$v_\varphi = v_\theta = 0, \ \rho = \text{const}. \tag{2.2}$$

With conditions (2.2), the equation of the conservation of mass has the form

$$\frac{\partial}{\partial r}(r^2 v_r) = 0. \tag{2.3}$$

The problem of the ejection of soil in the presence of axial symmetry contains two independent spatial variables and the time. A considerable simplification is achieved if, in the calculation of the problem, there remain only one spatial variable and the time. To this end, in the present work the equation of motion is integrated over the radius.

Let us obtain an integral form of Eq. (2.1) for an element of the continuous medium extending along the radius. We multiply Eq. (2.1) by $r^2 \sin\theta$ and integrate from the radius of the cavity r_1 to the radius of the surface of the ground r_2 (Fig. 1; the values of r_1 and r_2 depend on θ and t):

$$\int_{r_1}^{r_2} \rho \frac{\partial v_r}{\partial t} \sin\theta r^2 dr = \sin\theta \int_{r_1}^{r_2} \frac{\partial (r^2 \sigma_r)}{\partial r} dr + \frac{\partial}{\partial \theta}\left(\sin\theta \int_{r_1}^{r_2} \tau_{r\theta} r dr\right) - \sin\theta \int_{r_1}^{r_2} (\sigma_\theta + \sigma_\varphi) r dr - g\cos\theta \int_{r_1}^{r_2} \rho r^2 \sin\theta \, dr. \tag{2.4}$$

Here, on the right-hand part, in the second term the derivative is taken of the integral, and not of the expression under the integration sign. This result can be obtained by deriving Eq. (2.4) directly for an element of the medium extended along the radius. The designated integral is proportional to the tangential force acting on each lateral surface of the element. The total force acting on an element is proportional to the derivative of the integral.

The left-hand part of (2.4) is equal to the mass of medium in unit angle θ and angle φ, multiplied by the acceleration of the center of mass r_0,

$$\int_{r_1}^{r_2} \rho \frac{\partial v_r}{\partial t} \sin\theta \, r^2 dr = m \frac{\partial^2 r_0}{\partial t^2}; \tag{2.5}$$

$$m = \frac{1}{3} \rho \sin\theta \left(r_2^3 - r_1^3\right), \quad r_0 = \frac{3\left(r_2^4 - r_1^4\right)}{4\left(r_2^3 - r_1^3\right)}. \tag{2.6}$$

From the last relationship we obtain the velocity of the center of mass,

$$v_0 \equiv \frac{\partial r_0}{\partial t} = 3\frac{r_2^3 v_2 - r_1^3 v_1}{r_2^3 - r_1^3}, \quad v_1 = \frac{\partial r_1}{\partial t}, \quad v_2 = \frac{\partial r_2}{\partial t},$$

where v_1 is the velocity of the boundary of the cavity; v_2 is the velocity of the surface of the ground.

Integrating the equation of continuity (2.3), we obtain

$$r^2 v_r = r_1^2 v_1 = r_2^2 v_2 = \frac{1}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right) v_0. \tag{2.7}$$

The value of $r_0^2 v_0$ cannot be substituted into this equality, as was done erroneously in [10], since the center of mass is shifted over the particles of the medium, and the mass between r_1 and r_2 does not remain constant.

The stresses enter into Eq. (2.4) in such a manner that it is convenient to introduce the mean values of these quantities

$$x_0 = \int_{r_1}^{r_2} x r dr \bigg/ \int_{r_1}^{r_2} r dr, \quad x = \{p, \sigma_\theta, \sigma_\varphi, \tau_{r\theta}\}, \; x_0 = \{p_0 \sigma_{\theta 0}, \sigma_{\varphi 0}, \tau_0\}. \tag{2.8}$$

Using (2.5), (2.8), we transform (2.4) to the form

$$\frac{\partial v_0}{\partial t} = \frac{3}{2\rho\left(r_2^3 - r_1^3\right)\sin\theta} \frac{\partial}{\partial \theta}\left[\tau_0 \left(r_2^2 - r_1^2\right)\sin\theta\right] - \frac{3}{2\rho} \frac{r_1 + r_2}{r_1^2 + r_1 r_2 + r_2^2}\left(\sigma_{\theta 0} + \sigma_{\varphi 0}\right) - \frac{3\left(p_2 r_2^2 - p_1 r_1^2\right)}{r_2^3 - r_1^3} - g\cos\theta, \tag{2.9}$$

where p_1 is the pressure in the cavity; p_2 is the pressure at the surface of the ground. The values of r_1 and r_2 are determined from the algebraic relationships (2.6) using the values of r_0 and m.

The supplementary equations for finding τ_0, $\sigma_{\theta 0}$, $\sigma_{\varphi 0}$ depend on the model of the medium.

3. An Incompressible Medium with Coulomb Friction

During the process of the formation of the dome, the main mass of the ejected medium is subjected to considerable shear deformations, considerably exceeding those with which the theory of elasticity is still applicable. However, at the edges and the center of the dome, the shear deformations are small. Near the axis of symmetry, the elements are shifted only slightly with respect to one another; therefore, here the tangential stresses obey Hooke's law. Far from the axis of symmetry, at angles close to the horizontal, the elements of the medium themselves have small shifts, and the tangential stresses are also not great.

In the motion under consideration, the areas of slip are the lines θ=const. The tangential stresses in them are calculated in accordance with the relationships of the theory of elasticity, if they do not exceed the limiting value τ_*,

$$\tau_{r\theta}=\frac{\mu}{r}\frac{\partial u_r}{\partial\theta},\quad |\tau_{r\theta}|<\tau_*, \tag{3.1}$$

where μ is the shear modulus. We use relationship (3.1) in a form differentiated with respect to the time,

$$\frac{\partial\tau_{r\theta}}{\partial t}=\frac{\mu}{r}\frac{\partial v_r}{\partial\theta}. \tag{3.2}$$

In such a form, the stresses are calculated from the natural deformations, determined from the ratio of the shifts to the instantaneous dimensions, and to the original dimensions. In formula (3.2) this is taken into account by the fact that the quantity r is not differentiated.

We calculate the limiting stress using Coulomb's law,

$$\tau_*=c-k\sigma_\theta(\sigma_\theta<c/k),\ \tau_*=0(\sigma_\theta\geqslant c/k), \tag{3.3}$$

where c is the adhesion; k is the friction coefficient.

To calculate the mean values using formulas (2.8) we need to know the dependence of the stresses on the radius. The exact distribution depends on the radial motion of an element, consideration of which would greatly complicate the problem. Approximately, the normal stresses will be assumed equal to one another:

$$\sigma_r=\sigma_\theta=\sigma_\varphi=-p. \tag{3.4}$$

The dependence of the stress on the radius is assumed to be linear

$$\sigma_\varphi=\frac{p_2r_1-p_1r_2}{r_2-r_1}+\frac{p_1-p_2}{r_2-r_1}r. \tag{3.5}$$

In a homogeneous motionless medium, located in a field of gravity, the stresses depend on the distances in accordance with a linear law.

In another limiting case, that of a thin shell whose thickness is small in comparison with the radius of curvature, the stresses are distributed linearly over the thickness also with motion accompanied by acceleration (with velocities which are small compared with the speed of sound).

It is to be expected that the use of formula (3.5) for the case under consideration will not introduce any large errors, since it is used only for determining the mean tangential stresses. Using (3.4), (3.5), we obtain from (2.8)

$$p_0=\frac{p_1(2r_1+r_2)+p_2(r_1+2r_2)}{3(r_1+r_2)}.$$

Substituting the value fround for p_0 into (2.9) and simplifying, we obtain

$$\frac{\partial v_0}{\partial t}=\frac{3}{2\rho\left(r_2^3-r_1^3\right)\sin\theta}\frac{\partial}{\partial\theta}\left[\left(r_2^2-r_1^2\right)\tau_0\sin\theta\right]+\frac{p_1-p_2}{\rho(r_2-r_1)}-g\cos\theta. \tag{3.6}$$

We multiply (3.2) by r, integrate over the radius within the limits from r_1 to r_2, and, using (2.7), (2.8), we obtain

$$\frac{\partial\tau_0}{\partial t}=\frac{2\mu}{3r_1r_2(r_1+r_2)}\frac{\partial}{\partial\theta}\left[\left(r_1^2+r_1r_2+r_2^2\right)v_0\right]\quad(|\tau_0|<\tau_{*0}). \tag{3.7}$$

The limiting mean tangential stress τ_{*0} is obtained from (3.3),

$$\tau_{*0}=c+kp_0(p_0>-c/k),\ \tau_{*0}=0(p_0\leqslant -c/k). \tag{3.8}$$

The values of r_1 and r_2 are connected with r_0 by the algebraic formulas (2.6). The equation of the adiabatic curve connects the pressure in the cavity with its volume V_1. The volume is calculated from the values of $r_1(\theta)$,

$$\frac{dp_1}{p_1}=-\gamma\frac{dV_1}{V_1},\quad \frac{dV_1}{d\theta}=\frac{2\pi}{3}r_1^3\sin\theta. \tag{3.9}$$

The system of equations in partial derivatives (3.6), (3.7), together with relationships (3.9) and formulas (2.6), (3.8), describes the motion of a medium with explosion ejection, in a radial approximation.

With elastic tangential stresses, the system of equations (3.6), (3.7) is a system of hyperbolic type. The rate of propagation of perturbations in the plane t, θ, equal to the slope of the characteristic curve, is expressed in the form

$$\frac{d\theta}{dt} = \pm \sqrt{\frac{\mu}{r_1 r_2 \rho}}. \tag{3.10}$$

This rate depends on the distance from the end of the elements of mass to the origin of coordinates. With $r_1 \sim r_2$, from (3.10) we obtain the result that the linear velocity of the perturbations along the dome is equal to the velocity of shear waves in an unbounded medium. We note that, in the system of equations under consideration, there is a third family of characteristic curves θ=const.

4. Boundary and Initial Conditions

In seeking the solution of Eqs. (3.6)-(3.9), in the region $t \geq 0$, $0 \leq \theta \leq \theta_0$ ($\theta_0 < \pi/2$), the following boundary conditions are taken:

$$\tau|_{\theta=0} = 0; \qquad v_0|_{\theta=\theta_0} = 0.$$

The first condition is obtained from the symmetry of the problem, and the second reflects the absence of motion of the medium far from the explosion.

At the initial moment of time, it is necessary to give the velocity and the tangential stress, as well as the values of the radii of the cavity and the surface, as functions of the angle θ.

These data can be obtained either from calculations of spherically symmetrical explosions [11], or from experimental data on an underground explosion [2].

With calculations, the initial radius of the cavity is determined from its value at the moment when the wave reaches the free surface (the end of the spherically symmetrical motion in the medium) or at the moment when the reflected wave reaches the expanded cavity (the end of the spherical expansion of the cavity). From the experimental data, the initial radius can be determined from the channeling index in the given soil (from the ratio of the volume of the underground cavity to the weight of the charge) [2].

The initial velocity of the centers of mass is calculated from the value of the total kinetic energy of the medium E_k, taken on by it at the moment of time at which r_1 was determined:

$$v_0 = \frac{3}{r_2^3 - r_1^3}\left[\frac{r_1 r_2 (r_2 - r_1)}{2\pi\rho}\,\alpha E\right]^{1/2}, \tag{4.1}$$

where E is the energy of the explosion; α is a dimensionless quantity, equal to the ratio of the total kinetic energy of the medium at the selected moment of time to the total energy of the explosion.

Formula (4.1) was obtained under the assumption of the uniform distribution of the kinetic energy of the medium in the underground-explosion stage over all directions, taking account of the dependence of the velocity on the radius using formula (2.7). With $r_1 \sim r_2$, formula (4.1) is obvious.

The initial values of r_0 and r_2 are determined from relationships (2.6) using the values of r_1 and m. The latter value, with the explosion, at a depth H below the horizontal surface of the ground, of a charge of radius R, is found using the formula

$$m_0 = \frac{1}{3}\rho \sin\theta\,[(H/\cos\theta)^3 - R^3].$$

We note the special character of the solution with the above initial radii. The values of r_1 do not depend on the angle, while the values of r_2 rise considerably with an increase in θ. In this case, it follows from formula (3.10) that the characteristic curves with a plus sign approach each other, although the rate of approach falls over the course of time, due to the increase in the radii. Near the axis of symmetry, the points of intersection of the characteristic curves lie very far apart. In the middle part, the intersection of the characteristic curves can take place after a finite time. This intersection can be interpreted as the appearance of a discontinuity in the velocities, the displacements, and the tangential stresses along the surface, on one side of which there is an intense motion of the ejected medium, while on the other side there is practically no motion. It is more probable that the special characteristic under consideration will appear with a calculation of ejection in a rather strong medium, where condition (3.6), limiting the tangential stresses, is used in a smaller calculating region.

The initial pressure in the cavity is determined from the equation of state of the explosion products over a known volume of the cavity

$$p_1 = \frac{3(\gamma - 1)}{4\pi r_1^3}\beta E,$$

where γ is the adiabatic index of the explosion products; β is a dimensionless quantity, equal to the ratio of the internal energy of the gas in the cavity at a selected moment of time to the total energy of the explosion.

The tangential stresses at the initial moment of time in the calculations were assumed equal to zero since, with spherically symmetrical motion, they are absent.

5. Ballistic Scattering and Heaping

At the end of the second stage of the development of an ejection explosion the thickness of the dome becomes so small that the gas breaks through out of the cavity to the atmosphere, and the weakly connected medium forming the dome decomposes into individual pieces and starts to move along ballistic trajectories.

In the present work, the moment of breakdown is taken as the time of the ascent of the dome to a height equal to half the depth of the charge. Calculations have shown that the radius of the crater changes only slightly if the moment of breakdown is taken as a small height (one-third of the depth), or if the calculation is carried further down to atmospheric pressure in the cavity. In all these cases, the volume of the cavity increased considerably, and the pressure fell sharply and had only a slight effect on the pressure of the medium.

The ballistic scattering, with taking account of the resistance of the air (it is insignificant for large explosions), was calculated in accordance with a simple scheme. An element of mass will be ejected to the surface if it has sufficient kinetic energy to rise to the surface, with satisfaction of the condition

$$(v_0 \cos \theta)^2/2 \geqslant g(H - r_0 \cos \theta). \tag{5.1}$$

The horizontal flight distance, reckoned from the epicenter, is expressed in the form

$$L = r_0 \cos \theta + v_0 \sin \theta (v_0 \cos \theta + \sqrt{(v_0 \cos \theta)^2 - 2g(H - r_0 \cos \theta)})/g. \tag{5.2}$$

The dimension of the intermediate crater R_+ was determined from the maximal angle θ_+ for which the condition (5.1) $R_+ = H \tan \theta_+$ was satisfied.

If the angle θ_+ is greater than the angle of the internal friction of the medium, then the final dimension of the crater will be greater than the intermediate, due to slipping of the edges. In this case, a recalculation is made, starting from the conservation of the ejected mass. The slope of the sides of the final crater was taken equal to the angle of internal friction.

In this case, no account was taken of inertial sliding of the sides, which is not great for not too great depths.

The medium ejected to the surface of the ground forms a heap around the crater. Its height was determined from the condition of the conservation of mass

$$\int_{\theta}^{\theta_+} m d\theta = \int_{L_+}^{L} \rho h x dx \, (\partial L / \partial \theta < 0), \tag{5.3}$$

where h is the height of the heap; x is the distance from the epicenter; L_+ is the distance at which falls an element of mass, lying in the funnel at an angle of θ_+. If the inequality in (5.3) has the other sign, then the integration limits in one of the integrals must be exchanged.

Differentiating (5.3), we obtain

$$h = - m/(\rho_1 L \partial L/\partial \theta) \, (\partial L/\partial \theta < 0), \tag{5.4}$$

where ρ_1 is the density of the soil in the heap, which is generally less than the density of the medium in the solid ground. If the quantity $\partial L/\partial \theta$ changes sign, then two layers of soil will fall at exactly the same distance, and the total height of the heap will be determined by the sum of both layers.

Formulas (5.2), (5.4) define the dependence h(L) in terms of the parameter θ.

6. Results of Calculations

The equations in partial derivatives (3.6), (3.7) were approximated by a two-layer explicit difference scheme using centered differences with respect to space and time [12, 13].

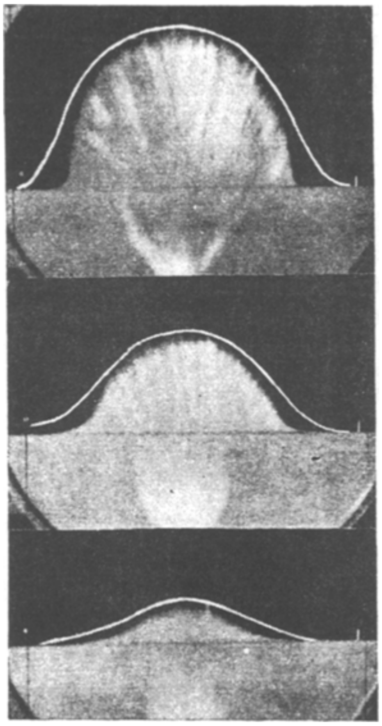

Fig. 2

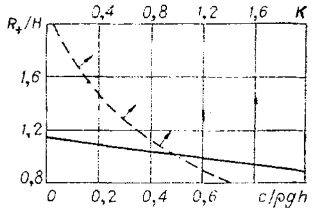

Fig. 3

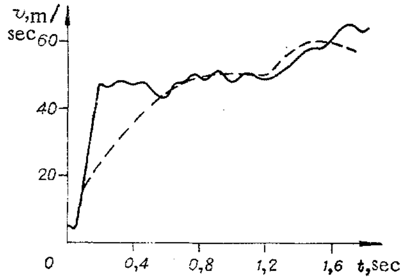

Fig. 4

TABLE 1

p_1, mm Hg	H, cm	r_1, cm	p_2, mm Hg	R_+/H exp.	R_+/H calc.
404	7,8	3,3	1,4	1,85	1,93
230	7,7	3,2	1,7	1,48	1,54
135	8	3,1	1,0	1,15	1,25
142	11,5	4,3	2,5	0,98	1,10

The stability condition was obtained from formula (3.10). Since the radii increased over the course of time, the permissible spacing with respect to time also increased. This circumstance promoted an acceleration of the calculation. If, in the new time layer, the stress exceeded the limiting value, the stresses were decreased in accordance with formula (3.9). A similar method was used in [3-6, 11] in the calculation of elastoplastic motions.

The difference scheme developed was used to calculate model experiments on the ejection of mass in a vacuum chamber [7]. The initial data were selected in accordance with experiment. The density of the sand was 1.52 g/cm^2, there was no adhesion, and the slope of the internal friction $k = 0.78$. The initial velocity was equal to zero. The initial pressures in an air bubble p_1, its radius r_1, the depth H, the pressure at the free surface, and the dimensions of the calculated and experimental craters are given in Table 1. The agreement is satisfactory.

Figure 2 shows the development of an ejection dome in a model experiment [7]. The solid line is a plot of the profile of the dome, obtained by calculation for exactly the same moments of time.

Figure 3 shows the values of the radii of the craters as a function of the friction coefficient (dashed line) and on the adhesion (solid line), calculated with the other parameters remaining unchanged. An increase in the friction or the adhesion decreases the radius of the craters formed, however, to a varying degree. The strongest effect is that of a change in the friction coefficient. In a medium which has not broken down, the region of the calculated changes in the maximal tangential stresses with a change in the adhesion or the friction coefficient is approximately identical. With ejection, the pressure near the cavity is considerably higher than the hydrostatic pressure; therefore, here the tangential stresses are considerably greater.

The method expounded can be used for the calculation of large ejection explosions. The solid line in Fig. 4 shows the rate of rise of the epicentric part of the dome as a function of the time, obtained in the Schooner experiment [14] (an underground nuclear experiment in the United States Trotyl equivalent 31 kilotons, depth 108 m), and calculated by the method proposed in the present article (dashed line). The agreement is satisfactory.

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QUESTIONS OF SIMILARITY AND THE SCATTERING OF WAVES IN VISCOPLASTIC MEDIA

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A study of plane waves in viscous media was made in [1-7]. A solution of the problem of the propagation of a wave set up by unsteady-state shock loading in a viscoelastic medium was obtained using an electronic computer in [6], and a solution in a viscoplastic medium in [1, 7]. In the latter case, different equations are introduced describing the behavior of the medium with loading and unloading, which leads to the formation of residual deformations. On the basis of the solutions of [1, 7], a finite-difference representation was constructed for the equations of motion in Lagrange variables, and for the sequence of differential equations determining the behavior of the medium. The method of "straight-through" calculation with pseudoviscosity was used. The introduction of the pseudoviscosity brings about the replacement of the shock fronts by regions of a continuous change in the parameters, which leads to additional difficulties in determination of the laws governing the washing-out of a shock wave and the scattering of waves. Below, the method of characteristic curves is used to obtain a solution to the problem of the propagation of a plane wave, set up by an unsteady-state shock load in a linear viscoplastic medium, corresponding to the model of [1]. It follows from the calculations that volumetric viscosity leads to scattering of the waves and to nonobservance of the condition of similarity. An increase by an order of magnitude in duration of a wave changes the rate of propagation of the maximum of the stresses, and the stresses themselves, by only a few percent. The values of the deformation and the velocity of the particles vary to a greater de-

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